ANOKA-RAMSEY COMMUNITY COLLEGE
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Summation

$$\sum_{k=\text{First index}}^{\text{Last index}} (k\text{th term})$$

Example:

$$\sum_{k=3}^{5} \frac{(-1)^k}{2k+1} = \frac{(-1)^3}{2(3)+1} + \frac{(-1)^4}{2(4)+1} + \frac{(-1)^5}{2(5)+1}$$

$$= -\frac{1}{7} + \frac{1}{9} - \frac{1}{11}$$

$$= -\frac{85}{693}$$

Summation Properties

Linearity:

$$\sum_{k=m}^{n} (ca_k + b_k) = c\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k$$

Example:

$$\sum_{k=11}^{20} (6k + k^2) = 6\sum_{k=11}^{20} k + \sum_{k=11}^{20} k^2$$

Sum Splitting:

$$\sum_{k=m}^{n} a_k = \sum_{k=1}^{n} a_k - \sum_{k=1}^{m-1} a_k$$

Example:

$$\sum_{k=6}^{20} k^2 = \sum_{k=1}^{20} k^2 - \sum_{k=1}^{5} k^2$$

Some Sums

Sum of a constant term:

$$\sum_{k=1}^{n} c = cn$$

Example:

$$\sum_{k=1}^{20} 6 = 6(20) = 120$$

Sum of the first n positive integers:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

Example:

$$\sum_{k=1}^{20} k = \frac{20(21)}{2} = 210$$

Sum of the first n perfect squares:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Example:

$$\sum_{k=1}^{20} k^2 = \frac{20(21)(41)}{6} = 70(41) = 2870$$

Sum of the first n perfect cubes:

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Example:

$$\sum_{k=1}^{20} k^3 = \left(\frac{20(21)}{2}\right)^2 = 210^2 = 44100$$