ARCC Tutoring Services | 763-433-1260
Rapids: Math Skills Center, Room L-127
Cambridge: ASC, Room D-208

Sequences & Series

Sequence: $a_1, a_2, a_3, \ldots$

Series: $a_1 + a_2 + a_3 + \cdots$

Arithmetic Sequences & Series

Common difference: d

***n*th term:**

$$\boldsymbol{a_n = a_1 + (n-1)d}$$

Example: Find the nth term of the sequence $10, 6, 2, -2, \ldots$

$$a_1 = 10, \qquad d = 6 - 10 = -4$$
$$a_n = 10 - 4(n-1) = -4n + 14$$

Recursive formula:

$$\boldsymbol{a_1 = a_1 \;\&\; a_n = a_{n-1} + d}$$

Example: List the first four terms of the sequence where $a_1 = 19$ & $a_n = a_{n-1} - 5$.

$$a_1 = 19 \qquad a_3 = a_2 - 5 = 9$$
$$a_2 = a_1 - 5 = 14 \qquad a_4 = a_3 - 5 = 4$$

Sum of the first *n* terms:

$$\boldsymbol{S_n = \frac{n}{2}(a_1 + a_n)}$$

Example:

$$7 + 13 + 19 + 25 + \cdots + 145 = ?$$
$$a_1 = 7 \quad d = 13 - 7 = 6$$
$$145 = 7 + 6(n-1) \Rightarrow n = 24$$
$$S_{24} = \tfrac{24}{2}(7 + 145) = 1824$$

Geometric Sequences & Series

Common ratio: r

***n*th term:**

$$\boldsymbol{a_n = a_1(r)^{n-1}}$$

Example: Find the nth term of the sequence $12, 4, \frac{4}{3}, \frac{4}{9}, \ldots$

$$a_1 = 12, r = \tfrac{4}{12} = \tfrac{1}{3} \Rightarrow a_n = 12\left(\tfrac{1}{3}\right)^{n-1}$$

Recursive formula:

$$\boldsymbol{a_1 = a_1 \;\&\; a_n = a_{n-1} \cdot r}$$

Example: List the first four terms of the sequence where $a_1 = 5$ and $a_n = -2a_{n-1}$.

$$a_1 = 5 \qquad a_3 = -2a_2 = 20$$
$$a_2 = -2a_1 = -10 \qquad a_4 = -2a_3 = -40$$

Sum of the first *n* terms:

$$\boldsymbol{S_n = \frac{a_1(1 - r^n)}{1 - r} \text{ if } r \neq 1}$$

Example: Find the sum of the first ten terms of the series $6 - 12 + 24 - 48 + \cdots$

$$a_1 = 6, \quad r = \tfrac{-12}{6} = -2$$
$$S_{10} = \frac{6(1 - (-2)^{10})}{1 - (-2)} = -2046$$

Sum of infinitely many terms:

$$\boldsymbol{S_\infty = \frac{a_1}{1 - r} \text{ if } |r| < 1}$$

(The series diverges if $|r| \geq 1$.)

Example: $8 + 2 + \frac{1}{2} + \frac{1}{8} + \cdots = ?$

$$a_1 = 8, \quad r = \frac{2}{8} = \frac{1}{4}$$
$$S_\infty = \frac{8}{1 - 1/4} = \frac{8}{3/4} = \frac{32}{3}$$