ARCC Tutoring Services | 763-433-1260
Rapids: Math Skills Center, Room L-127
Cambridge: ASC, Room D-208

Rational Functions

$\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are polynomials

Feature	How to Locate
Domain	where the denominator is <u>not</u> zero
Vertical Asymptotes	where the denominator is zero (after reducing)
Holes	The zeros of factors of the denominator which fully cancel out.
x-intercepts	where the numerator is zero (after reducing)
y-intercept	plug in $x = 0$, if possible

Relative degrees	Long-term Behavior
Denominator has bigger degree	Horizontal asymptote $y = 0$
Numerator & denominator have equal degrees	Horizontal asymptote $y = \frac{\text{leading coef of num}}{\text{leading coef of denom}}$
Numerator has bigger degree	No horizontal asymptote Oblique asymptote if deg num = deg denom + 1

Rational Function Examples

$$f(x) = \frac{x^2 + 9x - 36}{4x^2 - 17x + 15}$$

y-intercept:

$$f(0) = \frac{-36}{15} = -\frac{12}{5} \Rightarrow \left(0, -\frac{12}{5}\right)$$

Horizontal Asymptote: $y = 1/4$

Factor:

$$f(x) = \frac{(x-3)(x+12)}{(x-3)(4x-5)}$$

Domain:

$$(x-3)(4x-5) \neq 0$$
$$\{x \mid x \neq 3, 5/4\}$$

Reduce:

$$f(x) = \frac{x+12}{4x-5} \qquad \text{if } x \neq 3$$

Hole:

$$x - 3 = 0 \Rightarrow x = 3$$
$$\frac{3+12}{4(3)-5} = \frac{15}{7}$$
$$(3, 15/7)$$

x-intercept:

$$0 = x + 12 \Rightarrow (-12, 0)$$

Vertical Asymptote:

$$0 = 4x - 5 \Rightarrow x = 5/4$$

$$g(x) = \frac{3x + 2}{4x^2 + 7}$$

Horizontal Asymptote: $y = 0$

Vertical Asymptote: None