ARCC Tutoring Services | 763-433-1260
Rapids: Math Skills Center, Room L-127
Cambridge: ASC, Room D-208

Graphing Polynomials

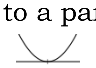
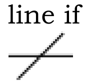
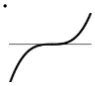
Standard form:

$$a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$$

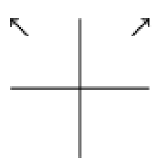
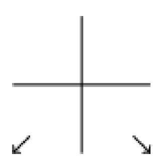
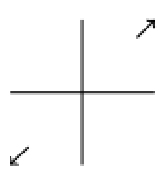
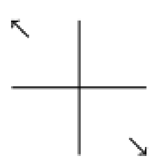
Domain: All real numbers

At most n real zeros.
At most $n-1$ turning points.

Real zeros/x-intercepts:

Multiplicity m of real zero r	Behavior of graph near x-intercept r
Even Multiplicity	**Touches** similar to a parabola: 
Odd Multiplicity	**Crosses** like a line if m=1:  but more like a cubic if $m > 1$: 

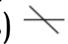
End behavior: For large $|x|$, the graph will resemble that of $y = a_n x^n$:

Degree n	Leading coefficient a_n	
	Positive	**Negative**
Even Degree		
Odd Degree		

Polynomial Examples

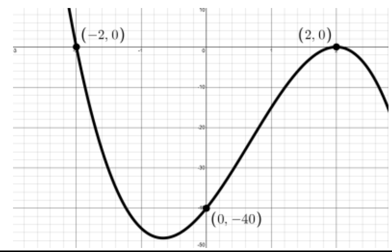
$$f(x) = -5x^3 + 10x^2 + 20x - 40$$

Factor: $f(x) = -5(x-2)^2(x+2)$

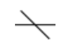
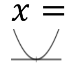
Zeros: $x = 2$ (mult. 2; touches) 
$x = -2$ (mult. 1; crosses) 

***y*-intercept**: $f(0) = -40 \Rightarrow (0, -40)$

End behavior: For large $|x|$, the function behaves like $y = -5x^3$. 



$$g(x) = 2(x-1)^3(x+4)(x^2-10)^4$$

Zeros: $x = 1$ (mult. 3; crosses) 
$x = -4$ (mult. 1; crosses) 
$x = \pm\sqrt{10}$ (each mult. 4; touches)  

***y*-intercept:**

$$g(0) = -80000 \Rightarrow (0, -80000)$$

End behavior: For large $|x|$, the function will behave like

$y = 2(x)^3(x)(x^2)^4 = 2x^{12}$