Mathematician_____

Algebra 2: Linear Programming Applications

1. A carpentry shop makes dinner tables and coffee tables. Each week the shop must complete at least 9 dinner tables and 13 coffee tables to be shipped to furniture stores. The shop can produce at most 30 dinner tables and coffee tables combined each week. If the shop sells dinner tables for $120 and coffee tables for $150, how many of each item should be produced for a maximum weekly income? What is the maximum weekly income?

2. A machine can produce either nuts or bolts, but not both at the same time. The machine can be used at most 8 hours a day. Furthermore, at most 6 hours a day can be used for making nuts and at most 5 hours a day can be used for making bolts. There is a $2 profit for each hour the machine makes nuts and a $3 profit for each hour the machine makes bolts. How many hours per day should the machine make each item in order to maximize profit? What is the maximum profit?

3. Mr. Wilson raises only pigs and goats, and this year he intends to raise no more than 16 animals. There is plenty of room in the pigpen, but lack of space limits the number of goats to 12. One other limitation is money: it costs $5/day to raise a pig and $2/day to raise a goat, and Mr. Wilson can spend only $50/day on the animals. If Mr. Wilson can make a profit of $17.50 per goat and $14.00 per pig, how many of each should he raise to maximize his profit? What is his maximum profit?

4. Mike is about to take a history test consisting of matching questions worth 10 points each and essay questions worth 25 points each. He is required to do at least 3 matching questions, but time restricts doing more than 12. Similarly, he must do at least 4 essays, but time restricts doing more than 15. If Mike can answer no more than 20 questions, how many of each type should he answer to maximize his score? What is the maximum score?

5. Cassie is about to take a math test that contains short-answer questions worth 4 points each and word problems worth 7 points each. She is required to do at least 5 short-answer questions, but no more than 10. She must also do at least 3 word problems but no more that 10. If she is required to do no more than 18 problems in total, how many of each type should she do to maximize her score? What is the maximum score?

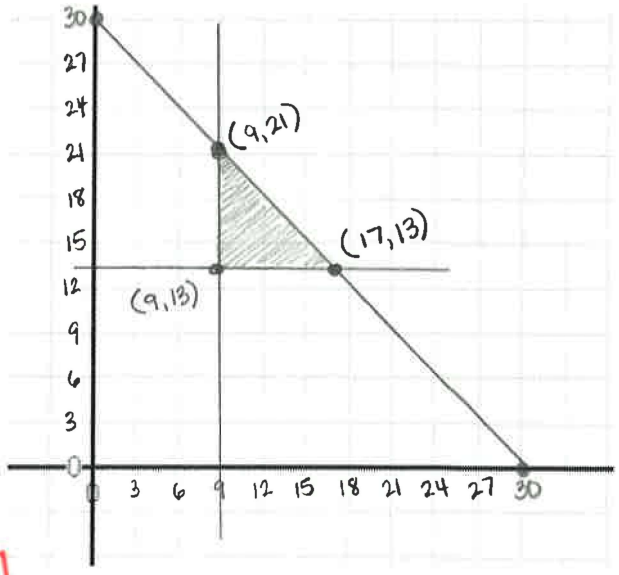
6. A sewing company makes 2 styles of girls' dresses: jumpers and smocks. It takes 10 minutes of cutting time and 30 minutes of sewing time to make a jumper while 30 minutes of cutting time and 15 minutes of sewing time are required to make a smock. At most, 20 hours each day can be allotted to cutting, while at most 15 hours each day can be used for sewing. For each jumper there is $5 profit and for each smock there is a $6.50 profit. If there is demand for all dresses produced, how many of each type should be made to earn the highest daily profit? What is the highest daily profit? (Hint: convert everything to minutes)

Mathematician Key

Algebra 2: Linear Programming Applications

1. A carpentry shop makes dinner tables and coffee tables. Each week the shop must complete at least 9 dinner tables and 13 coffee tables to be shipped to furniture stores. The shop can produce at most 30 dinner tables and coffee tables combined each week. If the shop sells dinner tables for $120 and coffee tables for $150, how many of each item should be produced for a maximum weekly income? What is the maximum weekly income?

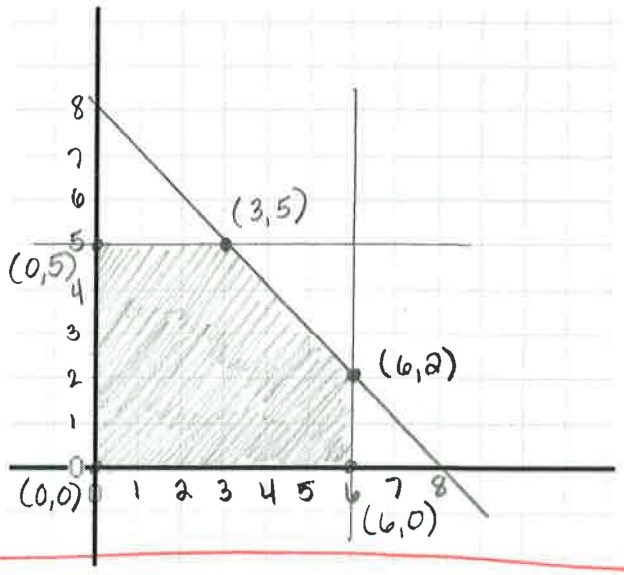
x = # of dinner tables
y = # of coffee tables
$x \geq 9$
$y \geq 13$
$x + y \leq 30$
$f(x,y) = 120x + 150y$
$f(9,13) = 120(9) + 150(13) = 3030$
$f(9,21) = 120(9) + 150(21) = 4230$
$f(17,13) = 120(17) + 150(13) = 3990$



There should be 9 dinner tables and 21 coffee tables for a maximum profit of $4230.

2. A machine can produce either nuts or bolts, but not both at the same time. The machine can be used at most 8 hours a day. Furthermore, at most 6 hours a day can be used for making nuts and at most 5 hours a day can be used for making bolts. There is a $2 profit for each hour the machine makes nuts and a $3 profit for each hour the machine makes bolts. How many hours per day should the machine make each item in order to maximize profit? What is the maximum profit?

x = # of hours for nuts
y = # of hours for bolts
$x + y \leq 8$
$x \leq 6 \quad x \geq 0$
$y \leq 5 \quad y \geq 0$
$f(x,y) = 2x + 3y$
$f(0,0) = 2(0) + 3(0) = 0$
$f(0,5) = 2(0) + 3(5) = 15$
$f(6,0) = 2(6) + 3(0) = 12$
$f(3,5) = 2(3) + 3(5) = 21$
$f(6,2) = 2(6) + 3(2) = 18$



There should be 3 hours for nuts and 5 hours for bolts with a maximum profit of $21.

3. Mr. Wilson raises only pigs and goats, and this year he intends to raise no more than 16 animals. There is plenty of room in the pigpen, but lack of space limits the number of goats to 12. One other limitation is money: it costs \$5/day to raise a pig and \$2/day to raise a goat, and Mr. Wilson can spend only \$50/day on the animals. If Mr. Wilson can make a profit of \$17.50 per goat and \$14.00 per pig, how many of each should he raise to maximize his profit? What is his maximum profit?

x = # of pigs

y = # of goats

$x + y \leq 16 \qquad x \geq 0$

$y \leq 12 \qquad y \geq 0$

$5x + 2y \leq 50 \longrightarrow 2y \leq -5x + 50$

$y \leq -\frac{5}{2}x + 25$

$f(x,y) = 14x + 17.5y$

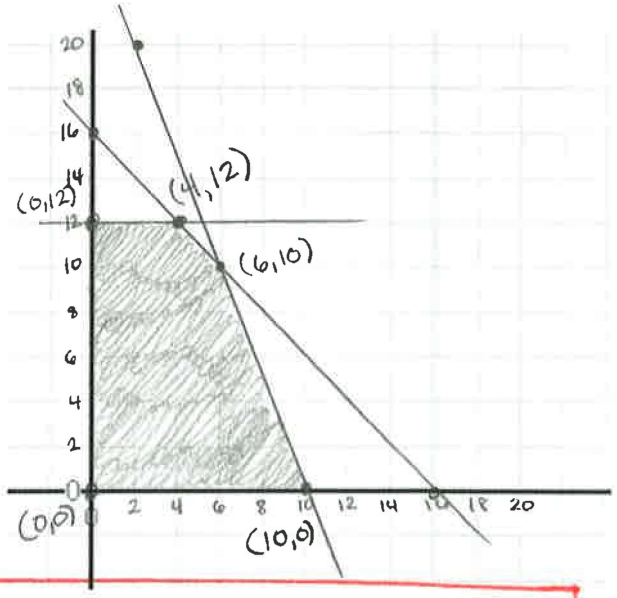
$f(0,0) = 14(0) + 17.5(0) = 0$

$f(0,12) = 14(0) + 17.5(12) = 210$

$f(10,0) = 14(10) + 17.5(0) = 140$

$f(6,10) = 14(6) + 17.5(10) = 259$

$f(4,12) = 14(4) + 17.5(12) = 266$



There should be 4 pigs and 12 goats for a maximum profit of \$266.

4. Mike is about to take a history test consisting of matching questions worth 10 points each and essay questions worth 25 points each. He is required to do at least 3 matching questions, but time restricts doing more than 12. Similarly, he must do at least 4 essays, but time restricts doing more than 15. If Mike can answer no more than 20 questions, how many of each type should he answer to maximize his score? What is the maximum score?

x = # of matching ?s

y = # of essay ?s

$3 \leq x \leq 12$

$4 \leq y \leq 15$

$x + y \leq 20$

$f(x,y) = 10x + 25y$

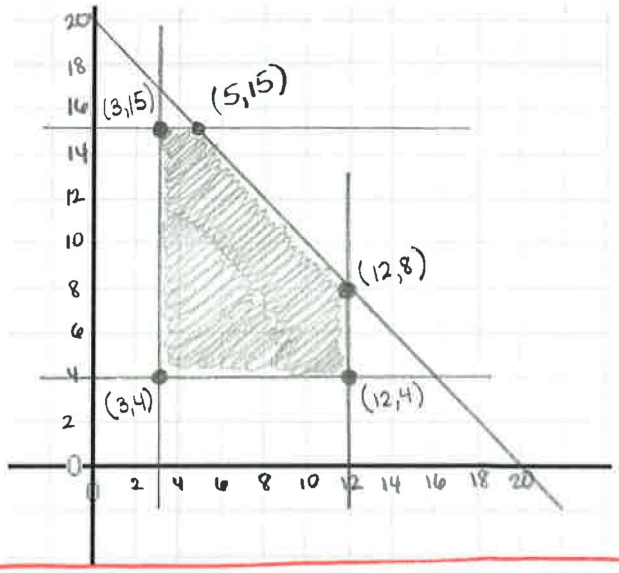
$f(3,15) = 10(3) + 25(15) = 405$

$f(3,4) = 10(3) + 25(4) = 130$

$f(12,4) = 10(12) + 25(4) = 220$

$f(12,8) = 10(12) + 25(8) = 320$

$f(5,15) = 10(5) + 25(15) = 425$



He should answer 5 matching and 15 essays for a maximum score of 425 points.

5. Cassie is about to take a math test that contains short-answer questions worth 4 points each and word problems worth 7 points each. She is required to do at least 5 short-answer questions, but no more than 10. She must also do at least 3 word problems but no more that 10. If she is required to do no more than 18 problems in total, how many of each type should she do to maximize her score? What is the maximum score?

x = # of short answers ?s

y = # of word problems

$5 \le x \le 10$

$3 \le y \le 10$

$x + y \le 18$

$f(x,y) = 4x + 7y$

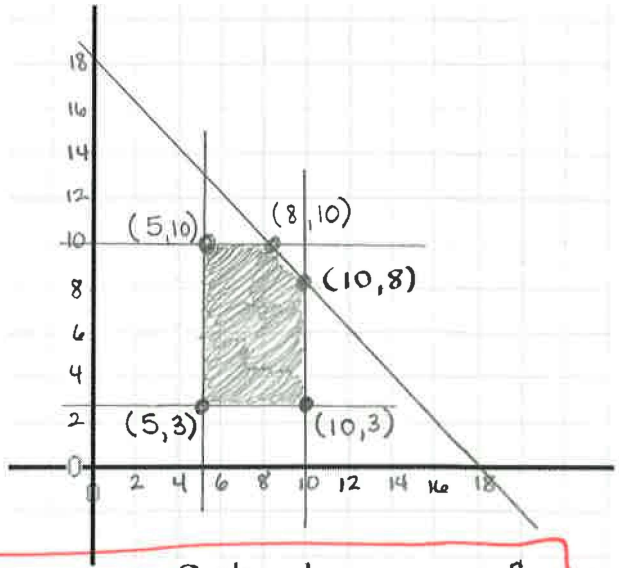
$f(5,10) = 4(5) + 7(10) = 90$

$f(5,3) = 4(5) + 7(3) = 41$

$f(10,3) = 4(10) + 7(3) = 61$

$f(10,8) = 4(10) + 7(8) = 96$

$f(8,10) = 4(8) + 7(10) = 102$



She should answer 8 short answer & 10 word problems for a maximum score of 102 points

6. A sewing company makes 2 styles of girls' dresses: jumpers and smocks. It takes 10 minutes of cutting time and 30 minutes of sewing time to make a jumper while 30 minutes of cutting time and 15 minutes of sewing time are required to make a smock. At most, 20 hours each day can be allotted to cutting, while at most 15 hours each day can be used for sewing. For each jumper there is $5 profit and for each smock there is a $6.50 profit. If there is demand for all dresses produced, how many of each type should be made to earn the highest daily profit? What is the highest daily profit? (Hint: convert everything to minutes)

x = # of jumpers

y = # of smocks

20hrs = 1200 min

15hrs = 900 min

cutting → $10x + 30y \le 1200$ → $30y \le -10x + 1200$

sewing → $30x + 15y \le 900$ → $y \le -\frac{1}{3}x + 40$

$x \ge 0$

$y \ge 0$

$15y \le -30x + 900$

$y \le -2x + 60$

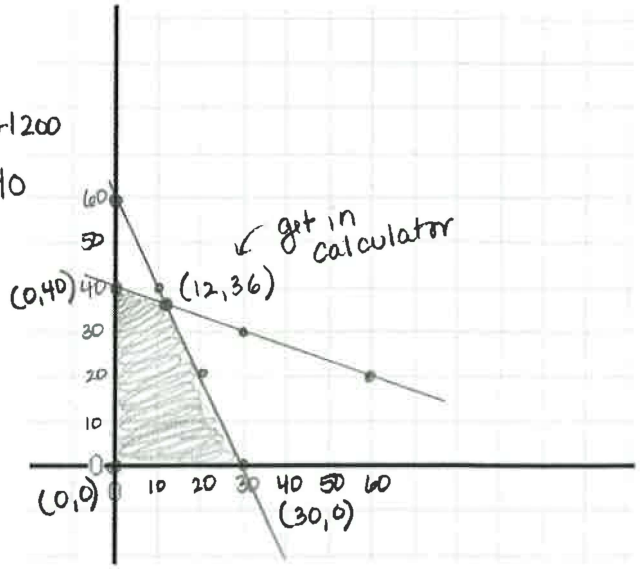
$f(x,y) = 5x + 6.5y$

$f(0,0) = 5(0) + 6.5(0) = 0$

$f(30,0) = 5(30) + 6.5(0) = 150$

$f(0,40) = 5(0) + 6.5(40) = 260$

$f(12,36) = 5(12) + 6.5(36) = 294$



They should make 12 jumpers and 36 smocks for a maximum profit of $294.