



## Complete the Square

General form of a:

Quadratic Equation:

$$ax^2 + bx + c = 0,$$

where  $a, b,$  &  $c$  are numbers.

How to generally solve

where  $a = 1$ :

$$x^2 + bx + c = 0$$

$$x^2 + bx + c - c = 0 - c$$

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

$$\left(x + \frac{b}{2}\right)^2 = -c + \left(\frac{b}{2}\right)^2$$

Then solve for  $x$ .

For Example:

$$x^2 + 6x - 27 = 0$$

Add 27 to both sides

$$x^2 + 6x = 27$$

Complete the square, add  $\left(\frac{b}{2}\right)^2$   
to both sides

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 27 + \left(\frac{6}{2}\right)^2$$

Simplify

$$x^2 + 6x + 9 = 36$$

Factor and Simplify

$$(x + 3)^2 = 36$$

Square root both sides

$$x + 3 = \pm\sqrt{36}$$

Simplify

$$x + 3 = \pm 6$$

Subtract 3 on both sides

$$x = -3 \pm 6 \text{ or } x = 3, -9$$

Final Results

How to generally solve  
where  $a \neq 1$  example:

$$2x^2 - 8x + 2 = 0$$

Subtract the 2 on both sides

$$2x^2 - 8x = -2$$

Divide everything by 2

$$x^2 - 4x = -1$$

Complete the square, add  $\left(\frac{b}{2}\right)^2$   
to both sides

$$\left(x^2 - 4x + \left(\frac{-4}{2}\right)^2\right) = -1 + \left(\frac{-4}{2}\right)^2$$

Simplify

$$(x^2 - 4x + 4) = -1 + 4$$

Factor and simplify

$$(x - 2)^2 = 3$$

Square root both sides

$$x - 2 = \pm\sqrt{3}$$

Add 2 to both sides

$$x = 2 \pm \sqrt{3}$$

Final two answers

Another example:

$$2x^2 + 3x - 5 = 0$$

$$2x^2 + 3x = 5$$

$$x^2 + \frac{3}{2}x = \frac{5}{2}$$

$$x^2 + \frac{3}{2}x + \left(\frac{3/2}{2}\right)^2 = \frac{5}{2} + \left(\frac{3/2}{2}\right)^2$$

$$x^2 + \frac{3}{2}x + \left(\frac{3}{4}\right)^2 = \frac{5}{2} + \left(\frac{3}{4}\right)^2$$

$$x^2 + \frac{3}{2}x + \frac{9}{16} = \frac{5}{2} + \frac{9}{16}$$

$$\left(x + \frac{3}{4}\right)^2 = \frac{40}{16} + \frac{9}{16} \Rightarrow \frac{49}{16}$$

$$x + \frac{3}{4} = \pm\sqrt{\frac{49}{16}} \Rightarrow \pm\frac{7}{4}$$

$$x = -\frac{3}{4} \pm \frac{7}{4} \text{ or } x = 1, -\frac{5}{2}$$

