MINNESOTA STATE
COLLEGES AND UNIVERSITIES
Intermediate
Algebra Sample
Questions

January 2017
Intermediate Algebra

The Minnesota State Colleges and Universities (MnSCU) Intermediate Algebra test contains 20 questions that measure proficiency in five content areas. The five content areas are as follows:

Linear Equations, Inequalities, Functions, and Systems—Topics covered in this category include:
- Algebraic operations with functions, literal equations, direct variation, and compound inequalities
- Solving systems of linear equations, linear inequalities, and linear functions
- Graphs of linear equations, inequalities, and functions
- Symbolic, graphical, and numerical representations of linear equations, inequalities, and functions
- Linear equations, inequalities, and functions with absolute values

Quadratic and Other Polynomial Expressions, Equations, and Functions—Topics covered in this category include:
- Algebraic operations involving quadratics and polynomials
- Solving quadratic equations
- Graphs of quadratic equations and functions
- Symbolic, graphical, and numerical representations of quadratic equations
- Operations with complex numbers

Expressions, Equations, and Functions Involving Powers, Roots, and Radicals—Topics covered in this category include:
- Algebraic operations involving rational and negative exponents
- Solving radical equations
- Graphs of simple square root functions

Rational and Exponential Expressions, Equations, and Functions—Topics covered in this category include:
- Algebraic operations involving rational expressions, equations, and functions
- Solving rational equations
- Exponential expressions, equations, and functions

Word Problems and Applications—Topics covered in this category include:
- Translating written phrases or sentences into algebraic expressions or equations
- Solving verbal problems in an algebraic context
Linear Equations, Inequalities, Functions, and Systems

For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.

1. If \( B = \frac{3}{2}x + d \), then \( x = \)

   A. \( \frac{2}{3}B - d \)
   B. \( \frac{2}{3}(B - d) \)
   C. \( \frac{3}{2}B - d \)
   D. \( \frac{3}{2}(B - d) \)

   \( y \geq 5x + 3 \)

2. If \( x \geq 2 \), then the inequality above is equivalent to which of the following?

   A. \( y \geq 10 \)
   B. \( y \leq 10 \)
   C. \( y \geq 13 \)
   D. \( y \leq 13 \)

3. If the equation \( y = \frac{3}{2}x - 1 \) were to be graphed in the \( xy \)-plane above, at what point would the graph intersect the line shown?

   A. \( \left( -\frac{4}{3}, 1 \right) \)
   B. \( \left( \frac{4}{3}, 1 \right) \)
   C. \( \left( 1, -\frac{4}{3} \right) \)
   D. \( \left( 1, \frac{4}{3} \right) \)

4. Which of the following shows the solution set of the system of inequalities above?

   \[
   \begin{align*}
   y &< x \\
   y &> -x
   \end{align*}
   \]

   A. 

   B. 

   C. 

   D.
5. If \( x + 3 \geq -1 \) and \( 2x + 5 \leq 1 \), then the graph of the solution set is

A. 
\[ \begin{array}{c|c|c|c|c|c|c} \hline -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 \\ \hline \end{array} \]

B. 
\[ \begin{array}{c|c|c|c|c|c|c} \hline -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline \end{array} \]

C. 
\[ \begin{array}{c|c|c|c|c|c|c} \hline -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 \\ \hline \end{array} \]

D. 
\[ \begin{array}{c|c|c|c|c|c|c} \hline -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 \\ \hline \end{array} \]

6. The equation \( y = mx + b \) is graphed in the \( xy \)-plane above. Which of the following must be equal to \( m \)?

A. \( \frac{t}{v} \)

B. \( -\frac{t}{v} \)

C. \( \frac{b}{v} \)

D. \( t - v \)

7. In the system of equations above, \( k \) is a constant. If the system has infinitely many solutions, what is the value of \( k \)?

A. \(-30\)

B. \(-\frac{5}{6}\)

C. \(\frac{5}{6}\)

D. \(30\)

8. For which of the following inequalities is the solution set equal to \( 4 \leq t \leq 10 \)?

A. \( |t + 10| \leq 4 \)

B. \( |t + 7| \leq 3 \)

C. \( |t - 4| \leq 10 \)

D. \( |t - 7| \leq 3 \)
Quadratic and Other Polynomial Expressions, Equations, and Functions

For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.

1. If \(12 - 3x^2 = 2y\) for all real numbers \(x\), then \(x^2 - 4 = \)
A. \(2y\)
B. \(\frac{2}{3}y\)
C. \(-\frac{2}{3}y\)
D. \(-\frac{3}{2}y\)

2. If \(x^4 + x^3 + x + 5 = x^4 + 25\), then \(x\) could be
A. \(-5\)
B. \(-4\)
C. \(2\)
D. \(5\)

3. If \(x^2 - 2 < \frac{7}{2}x\), which of the following must be true?
A. \(-4 < x < 2\)
B. \(-\frac{1}{2} < x < 4\)
C. \(x < -\frac{1}{2}\)
D. \(x > 4\)

4. In the \(xy\)-plane, if the point \((x, y)\) is on the \(x\)-axis and \(y = x^4 - 8x^2 - 9\), then \((x, y)\) could be
A. \((0, -3)\)
B. \((0, 1)\)
C. \((3, 0)\)
D. \((9, 0)\)

5. The parabola \(y = f(x)\) is graphed in the \(xy\)-plane above.
The graph of the parabola \(y = g(x)\) can be obtained by reflecting the graph of \(f\) across the \(y\)-axis. How many real roots does \(g(x)\) have?
A. Zero
B. One
C. Two
D. Three

6. The quadratic function \(y = f(x) = ax^2 + bx + c\) is graphed in the \(xy\)-plane above. The equation \(ax^2 + bx + c = dx\), where \(d\) is a constant, has
A. no real or complex solutions
B. exactly one real solution
C. two real solutions
D. one real solution and one complex solution

7. In the \(xy\)-plane, the graph of the parabola \(y = a(x - h)^2 + k\), where \(a, h,\) and \(k\) are constants, intersects the \(x\)-axis at two distinct points if and only if
A. \(h > 0\)
B. \(a < 0\)
C. the product of \(a\) and \(k\) is a negative number
D. the product of \(a\) and \(k\) is a positive number

8. Which of the following products, when multiplied out and simplified, does NOT result in a quadratic expression with real coefficients?
A. \((2x - \sqrt{3})(x + \sqrt{3})\)
B. \((ix)(1 - ix)\)
C. \((x - 8i)(x + 8i)\)
D. \((x + 2 - 3i)(x + 2 + 3i)\)
Expressions, Equations, and Functions Involving Powers, Roots, and Radicals

For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.

1. \(27^{\frac{4}{3}} = \)
   A. 9
   B. 36
   C. 81
   D. 243

2. If \(x\) is a positive number, \(\frac{\sqrt{2x^3}}{\sqrt{8x}} = \)
   A. \(\frac{x}{2}\)
   B. \(\frac{x}{4}\)
   C. \(\frac{x^2}{2}\)
   D. \(\frac{x^2}{4}\)

3. \(\sqrt{12} - 2\sqrt{3} = \)
   A. 0
   B. 2
   C. \(\sqrt{6}\)
   D. \(2\sqrt{3}\)

4. If \(x\) is a nonnegative number, then \(\sqrt{x^3 + 2x + 1} - 1 = \)
   A. \(x - 1\)
   B. \(x\)
   C. \(x + \sqrt{2x}\)
   D. \(x + \sqrt{2x - 1}\)

5. If \(f(x) = \sqrt{x - 5}\), for what value of \(x\) does \(f(x) = 10\)?
   A. 15
   B. 95
   C. 105
   D. 225

6. If \(\sqrt{a} + \sqrt{3} = \sqrt{48}\), then \(a = \)
   A. 16
   B. 24
   C. 27
   D. 45

7. For what value of \(x\) does \(x = \sqrt{(x-1)^2 + 60} - 1\)?
   A. 30
   B. 15
   C. 8
   D. 7

8. Which of the following could be the graph of \(y = -\sqrt{-x}\) in the \(xy\)-plane?
   A. 
   B. 
   C. 
   D.
Rational and Exponential Expressions, Equations, and Functions

For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.

1. \( \frac{7x}{x-3} - \frac{21}{x-3} = \)
   A. \( \frac{7}{x-3} \)
   B. \( \frac{7}{x} \)
   C. 7
   D. 0

2. \( \left( \frac{1}{a} - \frac{a}{b^2} \right) + \left( \frac{b^2 - a^2}{ab} \right) = \)
   A. 1
   B. \( \frac{1}{b} \)
   C. \( \frac{1-a}{b} \)
   D. \( \frac{a}{b(b^2-a^2)} \)

3. \( \frac{s}{3t-1} - \frac{s}{3t+1} = \)
   A. 0
   B. \( \frac{1}{3t^2-1} \)
   C. \( \frac{6st}{9t^2-1} \)
   D. \( \frac{2s}{9t^2-1} \)

4. There are two solutions to the equation above. What is the product of these two solutions?
   A. –36
   B. –9
   C. 13
   D. 36

5. How many different solutions does the equation \( \frac{3}{x(x-3)} + \frac{7}{x} = \frac{1}{x-3} \) have?
   A. None
   B. One
   C. Two
   D. More than two

6. If \( f(x) = \frac{2x}{x-1} \) and \( 1 < c < 2 \), which of the following could be \( f(c) \)?
   A. \( \frac{1}{2} \)
   B. \( \frac{5}{2} \)
   C. 4
   D. 6

7. If \( x + 8 = \frac{20}{x} \) and \( x > 0 \), which of the following is true?
   A. \( -1 < x < 1 \)
   B. \( \frac{1}{2} < x < 3 \)
   C. \( 2 < x < \frac{9}{2} \)
   D. \( 5 < x < 20 \)

8. In the \( xy \)-plane, which of the following points lies on the graph of \( y = x - \frac{1}{x} \)?
   A. \((-1, -2)\)
   B. \((0, -1)\)
   C. \((1, 0)\)
   D. \(\left(2, \frac{1}{2}\right)\)

\[ \frac{3}{x} - \frac{2}{x-1} = \frac{1}{12} \]
Word Problems and Applications

For each of the questions below, choose the best answer from the four choices given. You may use the paper you received as scratch paper.

1. To make a wood frame for a painting, Bella charges $3 per inch of the perimeter of the painting plus a fixed fee of $15 for the glass. If the square painting shown above has an area of $A$ square inches, how much would Bella charge, in dollars, to make a wood frame for the painting?

A. $3A + 15$
B. $3\sqrt{A} + 15$
C. $3\sqrt{A} + 15$
D. $12\sqrt{A} + 15$

2. A group of adults and children are at a puppet show. The number of children who attended was 8 more than the number of adults. If $c$ children are at the puppet show, which of the following represents the fraction of the people at the puppet show who are adults?

A. $\frac{c - 8}{c}$
B. $\frac{c}{2c - 8}$
C. $\frac{c - 8}{2c - 8}$
D. $\frac{c + 8}{2c + 8}$

3. A new factory opens and produces 5,000 units of a product the first day. Each day after the first, the factory produces 10,000 units of the product. Which of the following expresses the number of units of the product the factory produces the first $d$ days it is open?

A. $10,000d - 5,000$
B. $10,000d + 5,000$
C. $10,000(d + 1) + 5,000$
D. $10,000(2d) - 5,000$

4. Marjorie is renting an artist’s studio for a month at a cost of $800. During the month, if she makes $n$ pieces of pottery, her total cost, in dollars, for rent and materials will be $C(n) = 25n + 800$, and the function $g(n) = \frac{C(n)}{n}$ gives the cost, in dollars, per piece of making $n$ pieces of pottery. According to these functions, which of the following is true?

A. The total cost of making 100 pieces of pottery is the same as the total cost of making 120 pieces of pottery.
B. The total cost of making 100 pieces of pottery is greater than the total cost of making 120 pieces of pottery.
C. The cost per piece of making 100 pieces of pottery is the same as the cost per piece of making 120 pieces of pottery.
D. The cost per piece of making 100 pieces of pottery is greater than the cost per piece of making 120 pieces of pottery.
5. In rectangle $ABCD$ above, how much longer is diagonal $AC$ than side $AD$?

A. $\sqrt{64-w^2}$
B. $\sqrt{64+w^2}-w$
C. $\sqrt{64+w^2}-8$
D. $8+w-\sqrt{64+w^2}$

6. Working alone, Emma can paint a room in 1 hour, and working alone, Mason can paint the same room in 2 hours. If Emma and Mason work together, how long would it take for them to paint the room?

A. 40 minutes
B. 30 minutes
C. 25 minutes
D. 20 minutes

7. A ball was dropped from the top of a building, and the ball's height above the ground, in feet, $t$ seconds after it was dropped was $h(t) = -16t^2 + 100$. How many seconds after it was dropped did the ball hit the ground?

A. 1.6
B. 2.5
C. 6.25
D. 10

8. Ava bought some pens for $2 each and some pencils for $1 each. She bought 3 more pens than pencils and spent a total of $12. How many pencils did Ava buy?

A. 2
B. 3
C. 4
D. 5
### Answer Key

**Linear Equations, Inequalities, Functions, and Systems**

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Correct Answer</th>
<th>Rationale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>B</td>
<td>Choice (B) is correct. If ( B = \frac{3}{2}x + d ), then ( B - d = \frac{3}{2}x ). It follows that ( x = \frac{2}{3}(B - d) ).</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>Choice (C) is correct. If ( y \geq 5x + 3 ) and ( x \geq 2 ), then ( 5x \geq (5)(2) = 10 ). It follows that ( y \geq 5x + 3 \geq 10 + 3 = 13 ).</td>
</tr>
<tr>
<td>3</td>
<td>B</td>
<td>Choice (B) is correct. The line graphed in the figure has slope (-\frac{3}{2}) and has a (y)-intercept equal to 3. It follows that the equation of this line is ( y = -\frac{3}{2}x + 3 ). If the equation ( y = \frac{3}{2}x - 1 ) were to be graphed in the same (xy)-plane, then the point of intersection of the two lines would be the point ((x, y)) such that (x) and (y) satisfy both equations. If ( y = -\frac{3}{2}x + 3 ) and ( y = \frac{3}{2}x - 1 ) both hold, then (-\frac{3}{2}x + 3 = \frac{3}{2}x - 1), so (4 = 3x), and then (x = \frac{4}{3}). It follows that ( y = \left( -\frac{3}{2} \right) \left( \frac{4}{3} \right) + 3 = -2 + 3 = 1 ). Thus the graphs intersect at the point ((\frac{4}{3}, 1)).</td>
</tr>
<tr>
<td>4</td>
<td>C</td>
<td>Choice (C) is correct. The region in the (xy)-plane consisting of the points below the graph of the line (y = x) is the graph of (y &lt; x). The region in the (xy)-plane consisting of the points above the graph of the line (y = -x) is the graph of (y &gt; -x). The intersection of these two regions is the solution to the system of equalities (y &lt; x) and (y &gt; -x). This is the region shown in choice (C).</td>
</tr>
<tr>
<td>5</td>
<td>D</td>
<td>Choice (D) is correct. If (x + 3 \geq -1), then (x \geq -4). If (2x + 5 \leq 1), then (2x \leq -4) and (x \leq -2). It follows that if (x + 3 \geq -1) and (2x + 5 \leq 1), then (x \geq -4) and (x \leq -2). These inequalities describe the solution set graphed in choice (D).</td>
</tr>
<tr>
<td>6</td>
<td>B</td>
<td>Choice (B) is correct. The equation (y = mx + b) is in slope-intercept form, so the coefficient of (x), which is (m), is equal to the slope of the line. Also, the slope of the line is equal to (\frac{t - 0}{0 - v} = -\frac{t}{v}). Therefore, (m) must be equal to (\frac{-t}{v}).</td>
</tr>
<tr>
<td>7</td>
<td>D</td>
<td>Choice (D) is correct. If the system has infinitely many solutions, then the two equations are equivalent. That is, the set of ordered pairs ((x, y)) that satisfies (2x - 6y = 4) is the same set of ordered pairs that satisfies (10x - ky = 20). Equivalent equations have the property that one is a nonzero multiple of the other. Since (2x(5) = 10x), and (4(5) = 20), it follows that (-6y(5) = -ky). Therefore, (-30y = -ky) and (k = 30).</td>
</tr>
<tr>
<td>8</td>
<td>D</td>
<td>Choice (D) is correct. The inequality (4 \leq t \leq 10) is equivalent to the inequality (4 - 7 \leq t - 7 \leq 10 - 7), which is (-3 \leq t - 7 \leq 3). By the definition of absolute value, this is equivalent to (</td>
</tr>
</tbody>
</table>
Question Number | Correct Answer | Rationale
--- | --- | ---
1 | C | Choice (C) is correct. The equation $12 - 3x^2 = 2y$ can be rewritten as $3(4 - x^2) = 2y$. Multiplying both sides of the new equation by $-1$ and dividing by $3$ gives $x^2 - 4 = -\frac{2}{3}y$.

2 | A | Choice (A) is correct. Subtracting $x^4$ from both sides of the equation $x^4 + x^2 + x + 5 = x^4 + 25$ gives $x^2 + x + 5 = 25$, which is equivalent to $x^2 + x - 20 = 0$. Since $x^2 + x - 20 = (x + 5)(x - 4)$, it follows that the solutions of the equation $x^2 + x - 20 = 0$ are $-5$ and $4$. Therefore, of the choices given, $x$ can only be $-5$.

3 | B | Choice (B) is correct. The inequality $x^2 - 2 < -\frac{7}{2}x$ can be rewritten as $x^2 - \frac{7}{2}x - 2 < 0$, which is equivalent to $\left(x + \frac{1}{2}\right)(x - 4) < 0$. The solutions to the related equation $\left(x + \frac{1}{2}\right)(x - 4) = 0$ are $x = -\frac{1}{2}$ and $x = 4$. For $-\frac{1}{2} < x < 4$, the inequality $\left(x + \frac{1}{2}\right)(x - 4) < 0$ holds because $x + \frac{1}{2} > 0$ and $x - 4 < 0$. For all other real values of $x$, the product $\left(x + \frac{1}{2}\right)(x - 4)$ will be either positive, since both factors have the same sign, or zero. Therefore, $x$ must be between $-\frac{1}{2}$ and $4$. In other words, it must be true that $-\frac{1}{2} < x < 4$.

4 | C | Choice (C) is correct. Since the equation of the $x$-axis is $y = 0$, the point $(x, y)$ is of the form $(x, 0)$. The $x$-coordinate of the point can then be found by solving the equation $x^4 - 8x^2 - 9 = 0$, which can be rewritten as $(x^2 + 1)(x^2 - 9) = 0$. It follows that $x^2 = -1$ or $x^2 = 9$. The first of these equations has the two complex solutions $i$ and $-i$, and the second has the two real solutions $3$ and $-3$. Therefore, of the choices given, only $(3, 0)$ could be the point that satisfies the two conditions of the problem.

5 | A | Choice (A) is correct. The real roots of a function are the $x$-coordinates of the points where the graph of the function intersects the $x$-axis. Since the graph of $f$ does not intersect the $x$-axis and the graph of $g$ is obtained by reflecting the graph of $f$ across the $y$-axis, the graph of parabola $y = g(x)$ does not intersect the $x$-axis. Therefore, $g(x)$ has zero real roots.

6 | C | Choice (C) is correct. The graph of the function $y = dx$ is a nonvertical line that passes through the origin of the $xy$-plane. Any such line intersects the graph of the given parabola in two distinct points with different $x$-coordinates. Therefore, the equation $ax^2 + bx + c = dx$ has two real solutions.

7 | C | Choice (C) is correct. The graph of the parabola $y = a(x - h)^2 + k$ intersects the $x$-axis at two distinct points if and only if the equation $a(x - h)^2 + k = 0$ has two distinct real solutions. The equation $a(x - h)^2 + k = 0$ can be rewritten as $(x - h)^2 = -\frac{k}{a}$. The latter equation has two distinct real solutions if and only if $-\frac{k}{a} > 0$, which is equivalent to $\frac{k}{a} < 0$. If the quotient $\frac{k}{a}$ is negative, then the product of $a$ and $k$ will also be a negative number. It follows, then, that the equation $a(x - h)^2 + k = 0$ has two distinct real solutions if and only if the product of $a$ and $k$ is a negative number.

8 | B | Choice (B) is correct. Multiplying out the product $(ix)(1 - ix)$ gives $ix - x^2$. Since the imaginary number $i$ is the coefficient of $x$, not all the coefficients of the quadratic $ix + x^2$ are real numbers. All the other products, when multiplied out and simplified, result in quadratics with real coefficients: $\left(2x - \sqrt{3}\right)(x + \sqrt{3}) = 2x^2 + \sqrt{3}x - 3; (x - 8i)(x + 8i) = x^2 + 64; \ (x + 2 - 3i)(x + 2 + 3i) = x^2 + 4x + 13.$
### Expressions, Equations, and Functions Involving Powers, Roots, and Radicals

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<tr>
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<tr>
<td>1</td>
<td>C</td>
<td>Choice (C) is correct. Using the laws of exponents, (27^{\frac{4}{3}}) can be rewritten as ((27^{\frac{1}{3}})^4). Since the expression (27^{\frac{1}{3}}) is the cube root of 27, which is equal to 3, it follows that (27^{\frac{4}{3}} = 3^4 = 81).</td>
</tr>
<tr>
<td>2</td>
<td>A</td>
<td>Choice (A) is correct. The expression (\sqrt[4]{\frac{2x^3}{8x}}) can be rewritten as (\frac{\sqrt[4]{2x^3}}{\sqrt{8x}}), which is equal to (\sqrt{\frac{x^2}{4}}). Since (x) is a positive number, (\sqrt{\frac{x^2}{4}}) is equal to (\frac{x}{2}).</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>Choice (A) is correct. The expression (\sqrt{12}) is equal to (\sqrt{4 \times 3}), which is equal to (\sqrt{4} \times \sqrt{3}), or (2\sqrt{3}). Therefore, (\sqrt{12} - 2\sqrt{3} = 2\sqrt{3} - 2\sqrt{3} = 0).</td>
</tr>
<tr>
<td>4</td>
<td>B</td>
<td>Choice (B) is correct. Since ((x + 1)^2 = x^2 + 2x + 1), the expression (\sqrt{x^2 + 2x + 1} - 1) can be rewritten as (\sqrt{(x+1)^2} - 1). Since (x) is a nonnegative number, (x + 1) is also nonnegative, and so (\sqrt{(x+1)^2} - 1 = (x+1)-1 = x).</td>
</tr>
<tr>
<td>5</td>
<td>C</td>
<td>Choice (C) is correct. From the definition (f(x) = \sqrt{x - 5}), if (f(x) = 10), then (10 = \sqrt{x - 5}). Squaring both sides of (10 = \sqrt{x - 5}) gives (100 = x - 5), or (x = 105). Substituting (x = 105) in the definition for (f(x)) gives (f(105) = \sqrt{105 - 5} = \sqrt{100} = 10).</td>
</tr>
<tr>
<td>6</td>
<td>C</td>
<td>Choice (C) is correct. If (\sqrt{a} + \sqrt{3} = \sqrt{48}), then (\sqrt{a} = \sqrt{48} - \sqrt{3}). Since (\sqrt{48} = \sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3}), it follows that (\sqrt{a} = 4\sqrt{3} - \sqrt{3} = 3\sqrt{3} = \sqrt{9} \times \sqrt{3} = \sqrt{27}). Therefore, (a = 27).</td>
</tr>
<tr>
<td>7</td>
<td>B</td>
<td>Choice (B) is correct. The equation (x = \sqrt{(x-1)^2 + 60 - 1}) can be rewritten as (x+1 = \sqrt{(x-1)^2 + 60}). Squaring the quantities on both sides gives ((x+1)^2 = (x-1)^2 + 60). Expanding both sides of ((x+1)^2 = (x-1)^2 + 60) gives (x^2 + 2x + 1 = x^2 - 2x + 61), which simplifies to (4x = 60), or (x = 15). Substituting (x = 15) into the equation (x = \sqrt{(x-1)^2 + 60 - 1}) for (x) gives (15 = \sqrt{(15-1)^2 + 60 - 1}), or (15 = \sqrt{256 - 1}), which is true.</td>
</tr>
<tr>
<td>8</td>
<td>B</td>
<td>Choice (B) is correct. Each of the four graphs in the answer choices could be the graph of a radical function. Since (y = \sqrt{x}) is defined only for positive values of (x) and always yields a positive value for (y), the function (y = -\sqrt{x}) is defined only for negative values of (x) and always yields a negative value for (y). Hence the graph of (y = -\sqrt{x}) must lie within Quadrant IV of the (xy)-plane. Therefore, only the graph in choice (B) could be the graph of (y = -\sqrt{x}) in the (xy)-plane.</td>
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Rational and Exponential Expressions, Equations, and Functions

<table>
<thead>
<tr>
<th>Question Number</th>
<th>Correct Answer</th>
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<tbody>
<tr>
<td>1</td>
<td>C</td>
<td>Choice (C) is correct. The difference of the rational expressions ( \frac{7x}{x-3} - \frac{21}{x-3} ) can be rewritten as ( \frac{7x-21}{x-3} ). Factoring out 7 in ( \frac{7x-21}{x-3} ) gives ( \frac{7(x-3)}{x-3} = 7 ).</td>
</tr>
<tr>
<td>2</td>
<td>B</td>
<td>Choice (B) is correct. Dividing by a rational expression is equivalent to multiplying by its reciprocal. Hence, ( \left( \frac{1}{a} - \frac{a}{b^2} \right) + \left( \frac{b^2-a^2}{ab} \right) = \left( \frac{1}{a} - \frac{a}{b^2} \right) \left( \frac{ab}{b^2-a^2} \right) ). Since ( \frac{1}{a} - \frac{a}{b^2} = \frac{b^2-a^2}{ab^2} ), it follows that ( \frac{1}{a} - \frac{a}{b^2} \left( \frac{ab}{b^2-a^2} \right) ) can be rewritten as ( \left( \frac{b^2-a^2}{ab^2} \right) \left( \frac{ab}{b^2-a^2} \right) ), which simplifies to ( \frac{1}{b} ).</td>
</tr>
<tr>
<td>3</td>
<td>D</td>
<td>Choice (D) is correct. The difference of the rational expressions ( \frac{5}{3t+1} - \frac{s}{3t-1} ) is equivalent to ( \frac{s(3t+1)-s(3t-1)}{(3t-1)(3t+1)} ). Multiplying out the numerator and the denominator of the latter fraction gives ( \frac{3st+s-3st+s}{9t^2-1} ), which is equivalent to ( \frac{2s}{9t^2-1} ).</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
<td>Choice (D) is correct. The equation ( \frac{3}{x} - \frac{2}{x-1} = \frac{1}{12} ) can be rewritten as ( \frac{x-3}{x(x-1)} = \frac{1}{12} ). Cross multiplying in the latter equation gives ( 12(x-3) = x(x-1) ), which can be rewritten as ( x^2 - 13x + 36 = 0 ). Since the original equation ( \frac{3}{x} - \frac{2}{x-1} = \frac{1}{12} ) has two solutions, which are also solutions of the quadratic equation ( x^2 - 13x + 36 = 0 ), the product of these solutions is the constant term of ( x^2 - 13x + 36 = 0 ), which is 36. (In fact, the two solutions are ( x = 4 ) and ( x = 9 ).)</td>
</tr>
<tr>
<td>5</td>
<td>A</td>
<td>Choice (A) is correct. The equation ( \frac{3}{x(x-3)} + \frac{7}{x} = \frac{1}{x-3} ) can be rewritten as ( \frac{3}{x(x-3)} - \frac{1}{x-3} = \frac{7}{x} ). Factoring the expression ( \frac{1}{x-3} ) from the left-hand side of this equation yields ( \frac{1}{x-3} \left( \frac{3}{x} - 1 \right) = \frac{7}{x} ). Further simplification yields ( \frac{1}{x-3} \left( \frac{3-x}{x} \right) = \frac{7}{x} ), or ( \frac{1}{x} = \frac{7}{x} ). Since the equation ( \frac{1}{x} = \frac{7}{x} ) has no solutions, it follows that the original equation ( \frac{3}{x(x-3)} + \frac{7}{x} = \frac{1}{x-3} ) also has no solutions.</td>
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<tr>
<td>6</td>
<td>D</td>
<td>Choice (D) is correct. Since ( \frac{2x}{x-1} = 2 + \frac{2}{x-1} ), it follows that ( f(x) = 2 + \frac{2}{x-1} = 2 + 2 \left( \frac{1}{x-1} \right) ). If ( 1 &lt; c &lt; 2 ), then ( \frac{1}{c-1} &gt; 1 ) and so ( f(c) = 2 + \frac{2}{c-1} ) must be greater than 4. Of the choices given, the only value greater than 4 is 6. Therefore, only 6 could be equivalent to ( f(c) ). Let ( c = \frac{3}{2} ). Hence, ( f(c) = f \left( \frac{3}{2} \right) = 2 + \frac{2}{\left( \frac{3}{2} \right) - 1} = 6 ).</td>
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<td>7</td>
<td>B</td>
<td>Choice (B) is correct. Multiplying both sides of the equation $x + 8 = \frac{20}{x}$ by $x$ gives $x^2 + 8x = 20$, which is equivalent to $x^2 + 8x - 20 = 0$. The equation $x^2 + 8x - 20 = 0$ can be rewritten as $(x + 10)(x - 2) = 0$; therefore, the solutions to the equation are $x = -10$ and $x = 2$. Since $x &gt; 0$, it follows that $x = 2$. Therefore, of the choices given, only $\frac{1}{2} &lt; x &lt; 3$ is true.</td>
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<tr>
<td>8</td>
<td>C</td>
<td>Choice (C) is correct. Substituting $x = 1$ and $y = 0$ in the equation $y = x - \frac{1}{x}$ gives $0 = 1 - \frac{1}{1}$, which is true. Analogous substitutions for the other points given in the choices do not yield any true statements.</td>
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### Word Problems and Applications

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<td>1</td>
<td>D</td>
<td>Choice (D) is correct. Since the painting is a square and has area $A$ square inches, each side of the painting is $\sqrt{A}$ inches long. Therefore, the perimeter of the painting is $4\sqrt{A}$ inches. Hence, Bella would charge $3 per inch of the perimeter of the painting, or $12\sqrt{A}$ dollars, plus a fixed fee of $15 for the glass. Therefore, the total Bella would charge, in dollars, to make a wood frame for the painting would be $12\sqrt{A} + 15$.</td>
</tr>
<tr>
<td>2</td>
<td>C</td>
<td>Choice (C) is correct. Since $c$ children are at the puppet show and there are 8 more children than adults, the number of adults at the puppet show is $c - 8$. Hence, the total number of people — adults and children — at the puppet show is $c + (c - 8) = 2c - 8$. Since $c - 8$ of these 2$c - 8$ people are adults, the fraction of people at the puppet show who are adults is $\frac{c - 8}{2c - 8}$.</td>
</tr>
<tr>
<td>3</td>
<td>A</td>
<td>Choice (A) is correct. The first day it is open, the factory produces 5,000 units of the product. For each of the next $d - 1$ days it is open, the factory produces 10,000 units of the product, which gives a total of 10,000($d - 1$) units for these $d - 1$ days. Therefore, the total number of units of the product that the factory produces the first $d$ days it is open is $5,000 + 10,000(d - 1) = 5,000 + 10,000d - 10,0000 = 10,000d - 5,000$.</td>
</tr>
<tr>
<td>4</td>
<td>D</td>
<td>Choice (D) is correct. The function $g(n) = \frac{C(n)}{n} = \frac{25n + 800}{n} = 25 + \frac{800}{n}$ gives the cost, in dollars, per piece of making $n$ pieces of pottery. As $n$ takes on larger positive integer values, the value of $\frac{800}{n}$ gets smaller, and so the function $g(n) = 25 + \frac{800}{n}$ decreases as $n$, the number of pieces of pottery Marjorie makes, increases. It follows that the cost per piece of making 100 pieces of pottery is greater than the cost per piece of making 120 pieces of pottery.</td>
</tr>
<tr>
<td>5</td>
<td>B</td>
<td>Choice (B) is correct. Since $ABCD$ is a rectangle, triangle $ADC$ is a right triangle with legs $\overline{AD}$ and $\overline{DC}$ of lengths $w$ and 8, respectively. By the Pythagorean theorem, $\overline{AC}$ has length $\sqrt{8^2 + w^2} = \sqrt{64 + w^2}$. Therefore, $\overline{AC}$ is longer than $\overline{AD}$ by $\overline{AC} - \overline{AD} = \sqrt{64 + w^2} - w$.</td>
</tr>
<tr>
<td>6</td>
<td>A</td>
<td>Choice (A) is correct. Since Emma can paint the room in 1 hour, or 60 minutes, for each minute she works, Emma can paint $\frac{1}{60}$ of the room. Since Mason can paint the room in 2 hours, or 120 minutes, for each minute he works, Mason can paint $\frac{1}{120}$ of the room. Hence, for each minute Emma and Mason work together, they can paint $\frac{1}{60} + \frac{1}{120} = \frac{2}{120} + \frac{1}{120} = \frac{3}{120} = \frac{1}{40}$ of the room. Therefore, if Emma and Mason work together, it would take them 40 minutes to paint the room.</td>
</tr>
<tr>
<td>7</td>
<td>B</td>
<td>Choice (B) is correct. When the ball hits the ground, its height above the ground is 0 feet. Therefore, solving the equation $0 = h(t) = -16t^2 + 100$ for $t$ will give the number of seconds after being dropped that the ball hits the ground. Solving for $t$ yields $t^2 = \frac{100}{16} = 6.25$, or $t = \pm 2.5$. Since $t = -2.5$ seconds does not make sense in the setting of the problem, the solution is $t = 2.5$. Therefore, the ball hits the ground 2.5 seconds after it was dropped.</td>
</tr>
<tr>
<td>8</td>
<td>A</td>
<td>Choice (A) is correct. Let $n$ be the number of pencils Ava bought. Since Ava bought 3 more pens than pencils, she bought $n + 3$ pens. Since the pens cost $2 each and the pencils cost $1 each, Ava spent a total, in dollars, of $2(n + 3) + 1(n) = 3n + 6$. Since the total amount Ava spent was $12, it follows that $3n + 6 = 12$, which gives $n = 2$. Therefore, Ava bought 2 pencils.</td>
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