Math 1200 Final Exam Review
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Learner Outcomes: At the conclusion of the course, the student should be able to:

Prerequisite and other outcomes.

1. Evaluate the function $f(x)=x^2+10x$ at the indicated values. Find $f(3)$, $f(x+1)$, and $f(x-3)$

Solve problems 2 – 5 algebraically in the real number system. Answers should be exact.

2. $2x^2+6x=x-3$
3. $2-\frac{3}{m}=\frac{2}{m+2}$
4. $5|3-2b|-7=8$
5. $x^2-4x+2=0$
6. Use a graphing utility to approximate the real solutions of the equation rounded to two decimal places. All solutions lie between -10 and 10.
   $2x^3-x^2-2x+1=0$
7. Solve the inequality $-7<3-5x\le 8$. Express your answer in set-builder notation or interval notation, then graph the solution set.

Identify, transform, and/or produce the graph for a given function (including constant, linear, polynomial, parabolic, cubic, square root, absolute value, logarithmic, exponential and the rational function $y=1/x$).

8. Find the slope and *y*-intercept of the line and draw its graph. $4x-3y=15$
9. Sketch the line with slope -4 that passes through the point (4, -1). Find an equation for this line.
10. Find the maximum or minimum value of the function $f(x)=2+5x+5x^2$
11. Find the *x*- and *y*-intercepts of the graph of the equation $y=x^2-x-6$
12. Find a function whose graph is a parabola with vertex $(3,4)$ and that passes through the point $(1,-20)$.
13. Graph the polynomial in the given viewing rectangle. Find the coordinates of all local extrema. State each answer correct to two decimal places.
    $y=x^5+4x^2+6,\quad [-3,3]$ by $[-5,10]$
14. a. Graph the quadratic function $f(x)=x^2+2x-3$ by determining whether its graph opens up or down and by finding its vertex, axis of symmetry, *y*-intercept, and *x*-intercept(s), if any.
    b. Solve $f(x)\le 0$.
15. For the function $g(x)=3^x+2$:
    a. Graph *g* using transformations. State the domain, range, and horizontal asymptote of *g*.
    b. Determine the inverse of *g*. State the domain, range, and vertical asymptote of g^{-1}.
    c. On the same graph as *g*, graph g^{-1}.
16. Graph $f(x)=2x^2-4x+1$ by determining whether its graph opens up or down and by finding its vertex, axis of symmetry, *y*-intercept, and *x*-intercepts, if any.

17. Graph each function using the techniques of shifting, compressing or stretching, and reflections. Identify any intercepts on the graph. State the domain and, based on the graph, find the range.

a. $f(x) = |x| + 4$

b. $h(x) = \sqrt{x} - 1$

c. $f(x) = \sqrt{x+3}$

d. $h(x) = (x-1)^2 + 2$

18. Determine whether the equation defines *y* as a function of *x*. If it does, state *y* as a function of *x*.

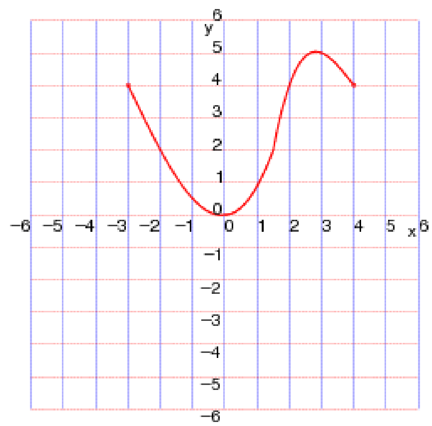
$(x + 7)^2 + y^2 = 49$

19. The graph of a function *h* is given.

a. Find $h(-2), h(0), h(2)$, and $h(3)$

b. Find the domain and range of *h*.

c. Find the values of *x* for which $h(x) = 4$



20. A function *f* is given. Draw the graph of the function on your calculator and use the graph to find the domain and range of *f*.

$f(x) = \sqrt{x+8}$

21. The function $f(x) = x^3 - 9x$ is given.

a. Find all the local **maximum** and **minimum** values of the function and the value of *x* at which each occurs. State each answer correct to two decimals.

b. Find the intervals on which the function is **increasing** and on which the function is **decreasing**. State each answer correct to two decimals.

22. Find the local maximum and minimum values of the function $f(x) = x^3 - 8x$ and the value of *x* at which each occurs. State each answer correct to two decimal places. (You may use a graphing calculator or graphing software.)

In problems 23 and 24, factor the polynomial and use the factored form to find the zeros. Then sketch the graph. (If a zero has multiplicity of two or higher, repeat its value that many times.)

23. $P(x) = x^3 - 2x^2 - 63x$

24. $P(x) = x^3 + 4x^2 - 9x - 36$

For problems 25 and 26, find the *x*- and *y*-intercepts of the rational function.

25. $r(x) = \dfrac{x+1}{x-2}$

26. $r(x) = \dfrac{x^2 + x - 20}{x^2}$

In problems 27 and 28, find the intercepts and asymptotes of each rational function.

27. $r(x) = \dfrac{3x+7}{-7x-2}$

28. $s(x)=\dfrac{3x-6}{x^2+2x-3}$

29. Find the vertical asymptote(s) of the function.

$$r(x)=\frac{x^2}{x-1}$$

30. Explain how the graph of *g* is obtained from the graph of *f*. What transformations are occurring?

a. $f(x)=x^2,\ g(x)=(x+8)^2$

b. $f(x)=x^2,\ g(x)=x^2+8$

A function *f* is given, and the indicated transformations are applied to its graph (in the given order). Write the equation for the final transformed graph.

31. $f(x)=x^3$, shift downward 3 units.

32. $f(x)=\sqrt[3]{x}$, shift 3 unit to the right.

33. $f(x)=x^2$, stretch vertically by a factor of 4, shift downward 3 units, and shift 5 units to the right.

34. Graph the function using transformations: $g(x)=\log_3(x-5)-3$

Identify, transform, and/or produce the graph of a circle.

35. Find the standard form $(x-h)+(y-k)^2=r^2$ of the equation of the circle whose center is $(3,4)$ and radius is $r=4$.

36. Find an equation of the circle that satisfies the given conditions. Give your answer using the form $(x-h)^2+(y-k)^2=r^2$. Center at the origin; passes through $(1,9)$

37. Find an equation of the circle that satisfies the given conditions. Give your answer using the form $(x-h)^2+(y-k)^2=r^2$. Circle lies in the first quadrant, tangent to both *x*- and *y*- axes; radius 3.

Find an equation of a line given sufficient information.

Find an equation of the line having the given characteristics. Express your answer using either the general form or the slope-intercept form of the equation of a line, whichever you prefer.

38. Find an equation of the line perpendicular to $3x-2y=7$ that contains the point $(1,5)$.

39. Find the equation of the line that is vertical and contains the point $(-3,4)$.

40. Find the equation of the line parallel to the line $x+y=2$ and contains the point $(1,-3)$.

41. Find an equation of the line that passes through (1, 10); slope 3

42. Find an equation of the line that passes through (1,-5); parallel to the line $x + 2y = 6$

43. Find the slope of the line through P and Q when $P=(2,4)$ and $Q=(-8,0)$.

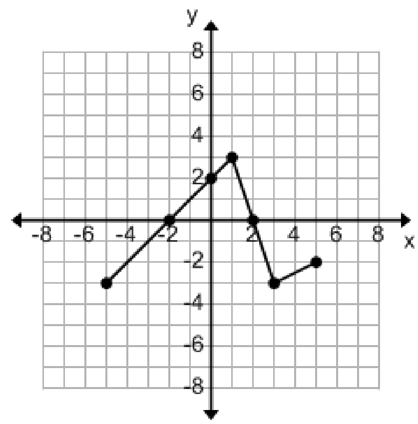
Translate an applied problem into an equation or inequality and provide a solution through algebraic manipulation.

44. Steve, a recent retiree, requires \$5000 per year in extra income. He has \$70,000 to invest and can invest in A-rated bonds paying 8% per year or in a certificate of deposit (CD) paying 5% per year. How much money should be invested in each to realize exactly \$5000 in interest per year?

45. Translate the following statement into a mathematical expression:
The perimeter p of a rectangle is the sum of two times the length l and two times the width w.

46. A coffee house has 20 pounds of a coffee that sells for \$4 per pound. How many pounds of coffee that sells for \$8 per pound should be mixed with the 20 pounds of \$4-per-pound coffee to obtain a blend that will for \$5 per pound? How much of the \$5-per-pound coffee is there to sell?

Interpret an expression, equation, or inequality by utilizing a graph, table, or diagram.



47. Using the graph of the function f:
  a. State the domain and the range of f.
  b. List the intercepts.
  c. Find $f(1)$.
  d. For what value(s) of x does $f(x)=-3$?
  e. Solve $f(x)<0$

Define a function along with its domain and range.

48. Find the domain of the function $f(x)=\sqrt{4-5x}$ and evaluate the function at $x=-1$.

For problems 49 – 52, find the domain of the function.

49. $f(x)=\dfrac{10}{x+1}$

50. $f(x)=\dfrac{x^4}{x^2+x-30}$

51. $f(x)=\sqrt{x^2-100}$

52. $f(x)=\sqrt{x}+\sqrt{6-x}$

Combine functions through the operations of addition, subtraction, multiplication, division, and composition.

53. Given $f(x)=\dfrac{x+2}{x-2}$ and $g(x)=2x+5$, find:
  a. $f\circ g$ and state its domain
  b. $g(f(-2))$
  c. $(f\circ g)(-2)$

54. Given the functions $f(x)=\sqrt{x+2}$ and $g(x)=2x^2+1$, find
  a. $f(g(2))$
  b. $(g\circ f)(-2)$
  c. $(f\circ f)(4)$

d. $(g \circ g)(-1)$

Given the following functions, find $f+g$, $f-g$, $f \cdot g$, and $\frac{f}{g}$. State the domain and range of each.

55. $f(x)=3x^2+x+1;\ g(x)=3x$

56. $f(x)=\frac{1}{x-3};\ \ g(x)=\frac{3}{x}$

Determine the inverse for a given function.

57. Find the inverse of $f(x)=\frac{2}{3x-5}$ and check your answer. State the domain and range of f and f^{-1}.

For problems 58 – 60, find the inverse of function *f*.

58. $f(x)=\frac{2}{3x-5}$

59. $f(x)=\frac{x-5}{x+5}$

60. $f(x)=1-x^3$

61. Draw the graph of *f* and use it to determine whether the function is one-to-one. (You may use a graphing calculator or graphing software.)

$$f(x)=4x^3+3x$$

Solve any equation of first or second degree.

62. Given that $f(x)=x^2+3x$ and $g(x)=5x+3$,
    a. solve $f(x)=g(x)$
    b. solve $f(x)>g(x)$

Find the real solutions of each equation.

63. $-5x+4=0$

64. $3x^2-5x-2=0$

65. $\sqrt[3]{1-x}=2$

Solve an exponential equation.

66. Express the equation $\log_{125}(5)=\frac{1}{3}$ in exponential form $a^y=x$.

67. Solve $3^x=243$ algebraically.

68. Solve $5^{x+2}=125$

69. Solve $4^{x-3}=8^{2x}$

Solve or work with logarithmic equations.

70. Express the equation $10^{04} = 0.0001$ in logarithmic form.

71. Evaluate the expressions.
   a. $\log_3\left(3^3\right)$
   b. $\log_3(9)$
   c. $\log_5(5)$

72. Solve $\log_b 16 = 2$ algebraically.

73. Solve $\log_5 x = 4$ algebraically.

74. Evaluate the expressions.
   a. $e^{\ln(\pi)}$
   b. $10^{\log(40)}$
   c. $10^{\log(90)}$

75. Use the definition of the logarithmic function to find *x*.
   a. $\log_4(x) = 2$
   b. $\log_2(0.5) = x$

Solve:

76. $8 - 2e^{-x} = 4$

77. $\log\left(x^2 + 3\right) = \log(x + 6)$

78. $7^{x+3} = e^x$

79. $\log_2(x - 4) + \log_2(x + 4) = 3$

80. Radioactive iodine is used by doctors as a tracer in diagnosing certain thyroid gland disorders. This type of iodine decays in such a way that the mass remaining after *t* days is given by the function $m(t) = 6e^{-0.083t}$ where *m* (*t*) is measured in grams.
   a. Find the mass at time *t* = 0.
   b. How much of the mass remains after 3 days? (Round your answer to 2 decimal places.)
   c. When will the mass be 3.96 grams?

81. If $3500 is borrowed at a rate of 11% interest per year, compounded quarterly, find the amount due at the end of the given number of years. (Round your answer to the nearest dollar.)
   a. 4 years
   b. 6 years
   c. When will the value be $10,359.56?

82. The present value of a sum of money is the amount that must be invested now, at a given rate of interest, to produce the desired amount at a later date. Find the present value of an investment if the desired amount is $10,000 and interest is paid at a rate of 6% per year, compounded semi-annually, for 3 years. (Round the answer to the nearest cent.)

83. Use the definition of the logarithmic function to find *x*.
   a. $\log_x(7) = 1/2$
   b. $\log_2(x) = 6$

84. A function $f(x)=\log_4(\log_6(x))$ is given.

a. Find the domain of the function *f*.

b. Find the inverse function of *f*.

85. Evaluate the expression $\log_5(40)-\log_5(72)+\log_5(225)$

For problems 86 – 88, use the Laws of Logarithms to expand the expression.

86. $\log_a\left(\frac{x^8}{yz^9}\right)$

87. $\log_7\left(\sqrt[6]{\frac{x-5}{x+5}}\right)$

88. $\log\left(\frac{10^x}{x(x^4+2)(x^6+6)}\right)$

For problems 89 and 90, use the Laws of Logarithms to combine the expressions.

89. $5\log_3(A)+3\log_3(B)-5\log_3(C)$

90. $-5\log(x)+\frac{1}{4}\log(x^2+1)+\log(x-1)$

91. Use the Change of Base Formula and a calculator to evaluate the logarithm $\log_9(12)$, correct to six decimal places. Use either natural or common logarithms.

92. Simplify $(\log_7 5)(\log_5 8)$.

93. Find the solution of the exponential equation $3^x=6^{2x+9}$, correct to four decimal places.

For problems 94 – 99, solve the equation for *x*.

94. $e^{2x}+1e^x-20=0$

95. $x^2 6^x-81(6^x)=0$

96. $\log(x-9)=2$

97. $\log_5 6+\log_5 x=\log_5 8+\log_5(x-5)$

98. $\log_3(x-3)+\log_3(x+3)=3$

99. $\log_2(\log_3(x))=1$

100. The population of a country has a relative growth rate of 5% per year. The government is trying to reduce the growth rate to 4%. The population in 1995 was approximately 110 million. Find the projected population for the year 2020 for the following conditions. Use the exponential growth equation.

a. The relative growth rate remains at 5% per year. (Round your answer to the nearest million.)

b. The relative growth rate is reduced to 4% per year. (Round your answer to the nearest million.)

101. The mass $m(t)$ remaining after t days from a 80 g sample of thorium-234 is given by the formula $m(t) = 80e^{-0.0275t}$.

a. How much of the sample will remain after 20 days? (Round your answer to the nearest tenth.)
b. After how long will only 30 g of the sample remain? (Round your answer to the nearest tenth.)
c. Find the half-life of thorium-234. (Round your answer to the nearest tenth.)

102. A hot bowl of soup is served at a dinner party. It starts to cool according to Newton's Law of Cooling so that its temperature at time t is given by the equation $T(t) = 64 + 142e^{-0.05t}$ where t is measured in minutes and T is measured in $°F$.

a. What is the initial temperature of the soup?
b. What is the temperature after 10 min?
c. After how long will the temperature be $100°F$?

Solve a system of linear equations in two or three variables.

For problems 103, 104, and 105, Solve the system, or show that it has no solution. If the system has infinitely many solutions, express them in the ordered pair form.

103. $\begin{cases} x - y = 1 \\ x + 3y = 5 \end{cases}$

104. $\begin{cases} -3x + 5y = 2 \\ 12x - 20y = 8 \end{cases}$

105. $\begin{cases} -\frac{1}{10}x + \frac{1}{2}y = 2 \\ 2x - 10y = -40 \end{cases}$

106. Find the complete solution of the linear system.

$$\begin{cases} x + y + z = -4 \\ x + 3y + 3z = 8 \\ 2x + y - z = -10 \end{cases}$$

107. Find the complete solution of the linear system. (If the system has infinitely many solutions, express your answer in terms of *k*. If the system has no solution, enter NONE for each answer.)

$$\begin{cases} y - 2z = -11 \\ 2x + 3y = 19 \\ -x - 2y + z = -4 \end{cases}$$

108. Find the complete solution of the linear system. (If the system has infinitely many solutions, express your answer in terms of *k*. If the system has no solution, enter NONE for each answer.)

$$\begin{cases} x + 2y - z = 0 \\ 2x + 3y - 4z = -12 \\ 3x + 6y - 3z = 3 \end{cases}$$

Solve a system of inequalities.

109. Graph the system of linear inequalities by hand. Verify your graph using a graphing utility.

$$\begin{cases} 2x + y \geq 0 \\ 2x + y \geq 2 \end{cases}$$

Solve a linear programming problem.

110. A banquet hall offers two types of tables for rent: 6-person rectangular tables at a cost of $28 each and 10-person round tables at a cost of $52 each. Kathleen would like to rent the hall for a wedding banquet and needs tables for 250 people. The room can have a maximum of 35 tables and the hall only has 15 rectangular tables available. How many of each type of table should be rented to minimize cost and what is the minimum cost?

State the definition of an infinite sequence.

111. Find the first four terms and the 100th term of the sequence $a_n = n + 7$

112. Find the first five terms of the given recursively defined sequence $a_n = 4(a_{n-1} - 4)$ and $a_1 = 5$

113. Find the *n*th term of a sequence whose first several terms are $2, 4, 8, 16, \ldots$

114. Find the *n*th term of a sequence whose first several terms are $2, 8, 14, 20, \ldots$

Work back and forth readily between expanded and closed forms of summation notation.

115. Find the sum $\sum_{k=1}^{5} k^2$

116. Find the sum $\sum_{i=1}^{9} \left[1 + (-1)^i\right]$

117. Write the sum $\sum_{k=6}^{9} k(k+5)$ without using sigma notation.

118. Write the sum, $5^2 + 6^2 + 7^2 + \ldots 14^2$, using sigma notation.

119. Find the *n*th term of the arithmetic sequence with given first term *a* and common difference *d*.
$a = 6,\ d = 2$
What is the 10^{th} term?

120. Determine the common difference, the fifth term, the *n*th term, and the 100th term of the arithmetic sequence.
$3, 7, 11, 15, \ldots$

121. The 12th term of an arithmetic sequence is 65, and the fifth term is 30. Find the 20th term.

122. Find the partial sum S_n of the arithmetic sequence that satisfies the given conditions $a_1 = 57, d = 11, n = 12$.

123. Find the sum of the infinite geometric series. $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$

Expand a binomial raised to natural number power less than six.

Expand each expression using the Binomial Theorem.

124. $(x-2)^6$

125. $(2x+3)^5$

Apply the definition(s) of the Fundamental Counting Principle, a permutation and a combination to counting problems as appropriate.

126. How many different arrangements are there of the letters in the word ROSE?

127. A professor has 10 similar problems to put on a test that has 3 problems. How many different tests can she design?

128. A woman has 5 blouses and 8 skirts. How many different outfits can she wear?

Apply the concepts of experiment, outcome, and sample space to a given model.

State the definition of probability of an event for a given sample space and apply such to simple problems.

129. Powerball is a multistate lottery in which 5 white balls from a drum with 53 balls and 1 red ball from a drum with 42 red balls are selected. For a $1 ticket, players get one chance at winning the jackpot by matching all 6 numbers. What is the probability of selecting the winning numbers on a $1 play?

Solve problems involving direct, inverse and joint variation.

130. Write a general formula to describe the variation, T varies jointly with the cube root of x and the square of d.